

### 1.3.13 Limits and properties of Bosonic Quantum Communication Channels

Bosonic Gaussian channels [1] are ubiquitous in physics. They arise whenever a harmonic system interacts linearly with a number of Bosonic modes which are inaccessible in principle or in practice [2,3]. They provide realistic noise models for a variety of quantum optical and solid state systems when treated as open quantum systems, including models for wave guides and quantum condensates. They play a fundamental role in characterizing the efficiency of a variety of tasks in continuous-variables quantum information processing [4], including quantum communication and cryptography. Most importantly, communication channels such as optical fibers can to a good approximation be described by Gaussian quantum channels.

In our analysis a complete analysis of multi-mode Bosonic Gaussian channels is proposed [5,6]. We clarify the structure of unitary dilations of general Gaussian channels involving any number of Bosonic modes and present a normal form. The maximum number of auxiliary modes that is needed is identified, including all rank deficient cases, and the specific role of additive classical noise is highlighted. By using this analysis, we derive a canonical matrix form of the noisy evolution of  $n$ -mode Bosonic Gaussian channels and of their weak complementary counterparts, based on a recent generalization of the normal mode decomposition for non-symmetric or locality constrained situations. It allows us to simplify the weak-degradability classification. Moreover, we investigate the structure of some singular multi-mode channels, like the additive classical noise channel that can be used to decompose a noisy channel in terms of a less noisy one in order to find new sets of maps with zero quantum capacity. Finally, the two-mode case is analyzed in detail. By exploiting the composition rules of two-mode maps and the fact that anti-degradable channels cannot be used to transfer quantum information, we identify sets of two-mode Bosonic channels with zero capacity.

We further present a formulation of the generalized minimal output entropy conjecture for Bosonic Gaussian channels [7]. It asserts that, for states with fixed input entropy, the minimal value of the output entropy of the channel (i.e. the minimal output entropy increment for fixed input entropy) is achieved by Gaussian states. In the case of centered channels (i.e. channels which do not add squeezing to the input state) this implies that the minimum is obtained by thermal (Gibbs) inputs. The conjecture is proved to be valid in some special cases.

A series of analytic upper bounds to the channel capacity  $C$  for transmission of classical information in these channels has been provided in [8]. In the practically relevant regimes of high noise and low transmissivity, by comparison with known lower bounds on  $C$ , our inequalities determine the value of the capacity up to corrections which are irrelevant for all practical purposes. Examples of such channels are radio communication, infrared or visible-wavelength free space channels. We also provide bounds to active channels that include amplification.

With increasing communication rates via quantum channels, memory effects become unavoidable whenever the use rate of the channel is comparable to the typical relaxation time of the channel environment (see Ref. [9] for a review on the subject – the paper been under consideration for publication by Rev. Mod. Phys.). In Refs. [10,11] we introduce a model of a Bosonic memory channel,

describing correlated noise effects in quantum-optical processes via attenuating or amplifying media. To study such a channel model, we make use of a proper set of collective field variables, which allows us to unravel the memory effects, mapping the  $n$ -fold concatenation of the memory channel to a unitarily equivalent, direct product of  $n$  single-mode Bosonic channels. We hence estimate the channel capacities by relying on known results for the memoryless setting. Our findings show that the model is characterized by two different regimes, in which the cross correlations induced by the noise among different channel uses are either exponentially enhanced or exponentially reduced.

Finally, still in the context of Bosonic Gaussian Channels in Ref. [12] we consider the problem of quantum communication mediated by an optical refocusing system, which is schematized as a thin lens with a finite pupil. This model captures the basic features of all those situations in which a signal is either refocused by a repeater for long distance communication, or it is focused on a detector prior to the information decoding process. Introducing a general method for linear optical systems, we compute the communication capacity of the refocusing apparatus. Although the finite extension of the pupil may cause loss of information, we show that the presence of the refocusing system can substantially enhance the rate of reliable communication with respect to the free-space propagation. An application of these scheme to the readout of a classical memory is presented in [13,14].

## References

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