1.3.15 Quantum Metrology

In classical estimation theory, the central limit theorem implies that the statistical error in a measurement outcome can be reduced by an amount proportional to n^(-1/2) by repeating the measures n times and then averaging. Using quantum effects, such as entanglement, it is often possible to do better, decreasing the error by an amount proportional to 1/n. Quantum metrology is the study of those quantum techniques that allow one to gain advantages over purely classical approaches [1].

In interferometry, sub-Heisenberg strategies claim to achieve a phase estimation error smaller than the inverse of the mean number of photons employed (Heisenberg bound). In a series of works [2,3] we show that one can achieve a comparable precision without performing any measurement, just using the large prior information that sub-Heisenberg strategies require. For uniform prior (i.e. no prior information), we prove that these strategies cannot achieve more than a fixed gain over Heisenberg-limited interferometry. Analogous results hold for arbitrary single-mode prior distributions. These results extend also beyond interferometry: the effective error in estimating any parameter is lower bounded by a quantity proportional to the inverse expectation value (above a ground state) of the generator of translations of the parameter. In Ref. [4] we analyze the how the sensitivity in optical interferometry is affected by losses during the signal propagation or at the detection stage. The optimal quantum states of the probing signals in the presence of loss were recently found. However, in many cases of practical interest, their associated accuracy is worse than the one obtainable without employing quantum resources (e.g. entanglement and squeezing) but neglecting the detector's loss. In our analysis we detailed an experiment that can reach the latter even in the presence of imperfect detectors: it employs a phase-sensitive amplification of the signals after the phase sensing, before the detection. We experimentally demonstrated the feasibility of a phase estimation experiment able to reach its optimal working regime. Since our method uses coherent states as input signals, it is a practical technique that can be used for high-sensitivity interferometry and, in contrast to the optimal strategies, does not require one to have an exact characterization of the loss beforehand. In Ref. [5] we studied the efficiency of quantum tomographic reconstruction where the system under investigation (quantum target) is indirectly monitored by looking at the state of a quantum probe that has been scattered off the target. In particular we focus on the state tomography of a qubit through a one-dimensional scattering of a probe qubit, with a Heisenberg-type interaction. Via direct evaluation of the associated quantum Cramer-Rao bounds, we compare the accuracy efficiency that one can get by adopting entanglement-assisted strategies with that achievable when entanglement resources are not available. Even though sub-shot noise accuracy levels are not attainable, we have shown that quantum correlations play a significant role in the estimation.

References
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