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Collective modes and ballistic expansion of a Fermi superfluid in the BCS-BEC crossover. Current experiments on ultracold atomic Fermi gases are focussed on superfluid states, and Bose-Einstein condensation of dimers has been achieved. A key tool for the manipulation of atomic gases is the use of a Feshbach resonance to vary the magnitude and sign of the coupling strength. Across the resonance the s-wave scattering length goes from large positive to large negative values, thus allowing exploration of the crossover from the Bardeen-Cooper-Schrieffer (BCS) state to the Bose-Einstein condensate (BEC) of bound-fermion pairs.

The frequencies of the collective modes are well known in both the BCS and the BEC limit. From non-mean-field perturbative estimates it has been conjectured that the frequency of the transverse breathing mode in a highly elongated trap should exhibit a non-trivial dependence on the scattering length [2]. With the aim to investigate this crucial point, we evaluate the frequencies of collective modes and the anisotropic expansion rate of a harmonically trapped Fermi superfluid at varying coupling strengths across a Feshbach resonance driving a BCS-BEC crossover [3]. We have used a microscopic mean-field description of the BCS-BEC crossover [4,5], which at zero temperature is believed to capture the essential physics in all regimes [6]. We calculate the equation of state and the density profiles of the gas under axially symmetric confinement with the help of a local density approximation, and use them to determine the collective mode frequencies and the expansion rate by means of a simple scaling assumption.

Our results do not use an interpolation scheme nor involve adjustable parameters, and show already at mean-field level non-monotonic behaviors across a Feshbach resonance. The theoretical results are compared in Fig. 1 with experimental data from two different groups [7,8], highlighting the degree of agreement between theory and experiments and the need for further experimental studies.

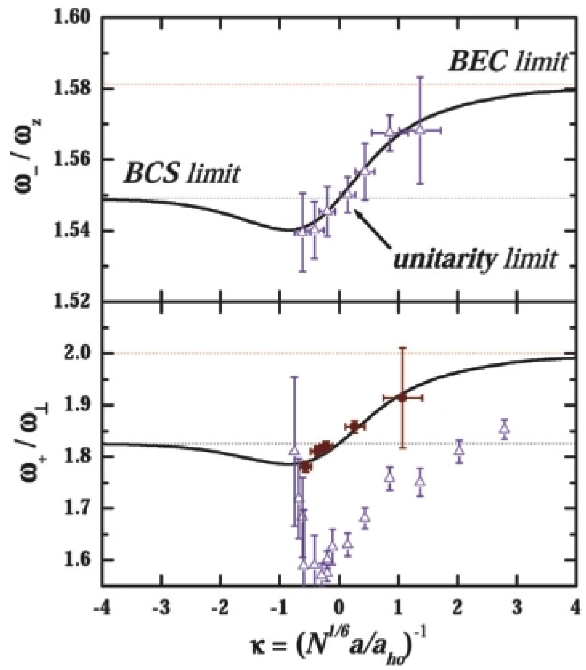

Fig. 1
Transverse (top) and axial (bottom) breathing modes in a trapped Fermi gas. The solid circles and empty triangles with error bars are the experimental results given by Kinast et al. [7] and by Bartenstein et al. [8].

Condensate localization in quasi-periodic structures

Solid-state-like systems such as a condensate or a quantum degenerate Fermi gas moving in a lattice can be realized by superposing periodic or quasi-periodic optical potentials to the atomic confinement. For instance, a quasi-onedimensional (1D) array of potential wells is created by the interference of two optical laser beams which counterpropagate. Such an optical lattice provides an almost ideal periodic potential and has allowed the study of Bloch and Josephson-like oscillations (see Ref. [1] and references therein).

We have studied the transport properties of condensed bosons through 1D multi-periodic and quasi-periodic arrays. We describe the system by means of a Bose-Hubbard Hamiltonian and use a scattering-matrix approach to evaluate the transmission coefficient of the condensate wavefunction travelling through the array under a constant external drive. When the lattice is modified by introducing further periodicities, in momentum space there exist several paths for Bloch oscillations and we have found that the number of condensed atoms leaving the lattice under the action of a constant force reflects the interference between the different paths [9,10] (see Fig. 2). On the other hand, in the presence of quasi-periodicity the spectrum becomes fragmented, the simple picture for Bloch oscillations breaks down and localization effects appear [10,11]. In Fig. 3 we show a schematic drawing of a set-up of optical lasers that would create an atomic Fibonacci wave guide. Two pairs of counterpropagating laser beams create a square optical lattice. The projection of this lattice on a line, at an angle (relative to the lattice) whose tangent corresponds to the golden ratio, creates a quasi-periodic sequence of bond lengths, and hence of hopping energies, which obey the Fibonacci rule [11]. The atoms can be made to travel along the sequence by pointing a hollow beam along this direction. Only for particular values of the external force, which could be controlled by varying the orientation of the whole set-up with respect to the vertical axis, the condensed atoms can be extracted from the array (see Fig. 2).

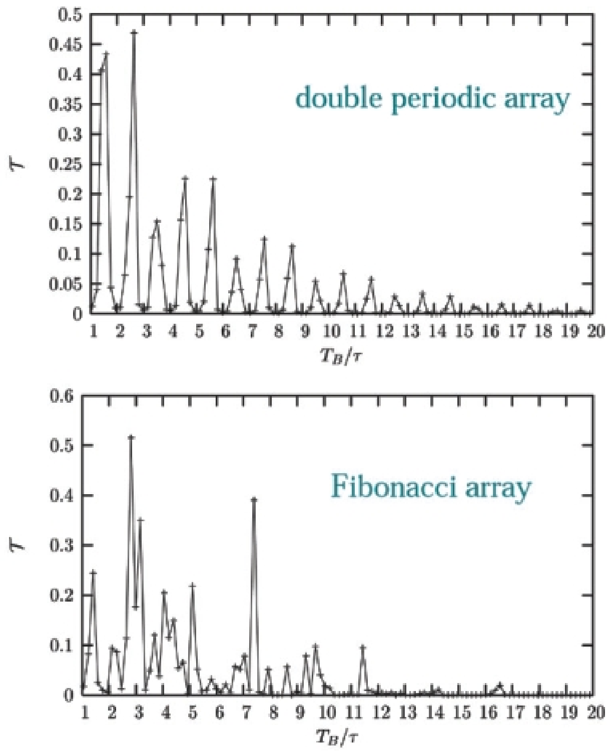

Fig. 2
Condensate transmittivity from period doubling (top) and through a Fibonacci chain (bottom), as a function of the ratio between the Bloch period T_B and the characteristic time t related to tunnel through the gap in the double period lattice.

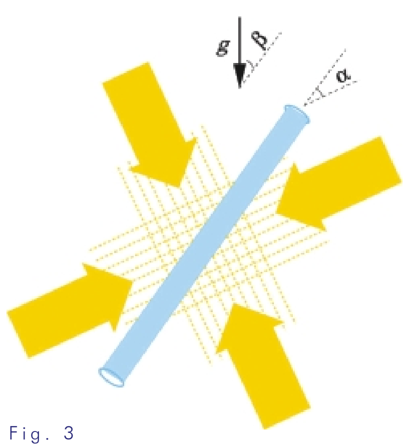

Fig. 3

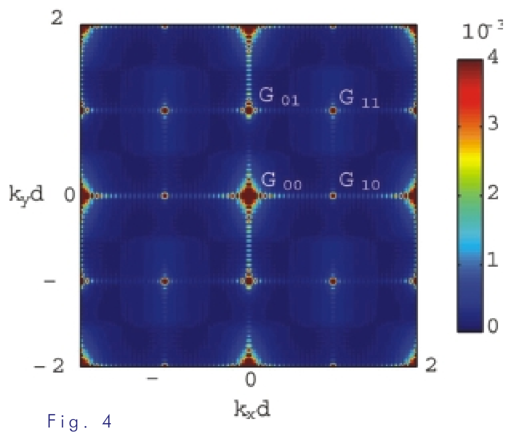

Fig. 4

Fully frustrated cold atoms

Fully frustrated Josephson Junction arrays (FF-JJA's) [12,13] exhibit a compound phase transition in which an Ising transition associated with discrete broken translational symmetry and a Berezinskii-Kosterlitz-Thouless (BKT) transition associated with phase coherence occur nearly simultaneously.

In a recent work [14] we have proposed that ultracold atoms be used to study the incompletely understood phase transitions that occur in FF-JJA's. As a model for ultracold atoms hopping in a 2D optical lattice we have used the Quantum Phase Model [12] with hopping energy E_J and on-site Hubbard repulsion U. We have calculated the momentum distribution function $n_f(\mathbf{k})$ within a self-consistent "spin-wave" approximation, in the presence of both quantum and thermal fluctuations.

A typical result for a finite lattice with $\mathbf{N}_s$=1296 sites with periodic boundary conditions is reported in Fig. 4. In the broken translation symmetry state $n_f(\mathbf{k})$ is non-zero at superlattice reciprocal lattice vectors $\mathbf{G}_{n,m}$= p(n,m)/d, d being the lattice constant. The expected BKT peaks at $\mathbf{k}$=$\mathbf{G}_{2n,2m}$ associated with the gauge U(1) symmetry are evident but the discrete Z_2 Ising-like symmetry induces non-zero satellites at $\mathbf{k}$=$\mathbf{G}_{1,0}$, $\mathbf{G}_{0,1}$ and $\mathbf{G}_{1,1}$. These are a sharp manifestation of the broken discrete translational symmetry and would be absent in an unfrustrated system.

So, both U(1) and Z_2 orders can be studied by momentum-distribution measurements.

Fig. 3
Schematic representation of a five laser-beam configuration to create a quasi-one-dimensional Fibonacci array for an atomic gas. Four beams generate a square optical lattice and a hollow beam confines the gas to a strip with slope a relative to an axis of the lattice, whose tangent corresponds to the golden ratio. The angle b between the hollow beam and the vertical direction determines the driving force as F=mg cos(b).

Fig. 4
Properly normalized momentum distribution function $n_f(\mathbf{k})$ [14] for FF cold bosons in a 2D lattice with a modulation parameter a=0.5 [15] as a function of the continuous variable **k**d , [-2p,2p]´ [-2p,2p]. In this case the thermal energy is k_BT=0.242E_J and the Hubbard repulsion is U=0.1 E_J.

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