

### 1.3.8 Quantum Thermodynamics: from Quantum Thermal Machines to Quantum Batteries

*Quantum thermodynamics [1,2] has emerged both as a field of fundamental interest, and as a potential candidate to improve the performance of thermal machines. Our activities span from theoretical characterization of the thermodynamics processes at the quantum level, to the development of control techniques to improve the performance of thermal processes (thermalization, cooling, etc), to the analysis of nanodevices for the storage of energy (quantum-battery models).*

In Ref. [3] two different models of optomechanical systems where a temperature gradient between two radiation baths is exploited for inducing self-sustained coherent oscillations of a mechanical resonator was studied. Viewed from a thermodynamic perspective, such systems represent quantum instances of self-contained thermal machines converting heat into a periodic mechanical motion and thus they can be interpreted as nano-scale analogues of macroscopic piston engines. Our models are potentially suitable for testing fundamental aspects of quantum thermodynamics in the laboratory and for applications in energy efficient nanotechnology.

In Ref. [6] we studied the dynamics of heat flux in the thermalization process of a pair of identical quantum system that interact dissipatively with a reservoir in a *cascaded* fashion. Despite the open dynamics of the bipartite system is globally Lindbladian, one of the subsystems "sees" the reservoir in a state modified by the interaction with the other subsystem and hence it undergoes a non-Markovian dynamics.

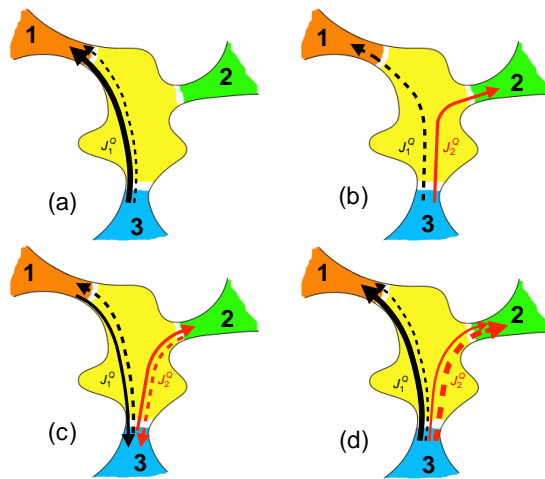
The use of quantum control techniques to improve the performances of thermal machines has been studied in a series of papers. Specifically, in Ref. [4] we consider the problem of time optimal control of a continuous bosonic quantum system subject to the action of a Markovian dissipation. In Ref. [7] optimal control strategies were analyzed to optimize the relaxation rate towards the fixed point of a quantum system in the presence of a non-Markovian dissipative bath. Contrary to naive expectations that suggest that memory effects might be exploited to improve optimal control effectiveness, non-Markovian effects influence the optimal strategy in a non-trivial way: we presented a necessary condition to be satisfied so that the effectiveness of optimal control is enhanced by non-Markovianity subject to suitable unitary controls. For illustration, we specialize our findings for the case of the dynamics of single qubit amplitude damping channels. The optimal control strategy presented here can be used to implement optimal cooling processes in quantum technologies and may have implications in quantum thermodynamics when assessing the efficiency of thermal micro-machines. In Ref. [11] an optimal process for probabilistic work extraction beyond the second law was introduced and in Ref. [12] an experimental verification of the scheme was produced with single-electron devices. In Ref. [20] we studied how to achieve the ultimate power in the simplest, yet non-trivial, model of a thermal machine, namely a two-level quantum system coupled to two thermal baths. Without making any prior assumption on the protocol, via optimal control we show that, regardless of the microscopic details and of the operating mode of the thermal machine, the maximum power is universally achieved by a fast Otto-cycle like structure in which the controls are rapidly switched between two extremal

values. A closed formula for the maximum power is derived, and finite-speed effects are discussed. We also analyzed the associated efficiency at maximum power (EMP) showing that, contrary to universal results derived in the slow-driving regime, it can approach Carnot's efficiency, no other universal bounds being allowed. In Ref. [21] non-Markovian effect on the efficiency of thermal machines were analyzed.

In Refs. [13] we developed a perturbation theory of quantum (and classical) master equations with slowly varying parameters, applicable to systems which are externally controlled on a time scale much longer than their characteristic relaxation time. This technique was used to study finite-time isothermal processes in which, differently from quasi-static transformations, the state of the system is not able to continuously relax to the equilibrium ensemble. Within first order in the perturbation expansion, we identify a general formula for the efficiency at maximum power of a finite-time Carnot engine. We also clarify under which assumptions and in which limit one can recover previous phenomenological results as, for example, the Curzon-Ahlborn efficiency. In Ref. [14] instead we applied advanced methods of control theory to open quantum systems and we determine finite-time processes which are optimal with respect to thermodynamic performances. General properties and necessary conditions characterizing optimal drivings were derived, obtaining bang-bang type solutions corresponding to control strategies switching between adiabatic and isothermal transformations. A direct application of these results is the maximization of the work produced by a generic quantum heat engine, where we show that the maximum power is directly linked to a particular conserved quantity naturally emerging from the control problem. Finally, we used our general approach to the specific case of a two level system, which can be put in contact with two different baths at fixed temperatures, identifying the processes which minimize heat dissipation. Moreover, we explicitly solved the optimization problem for a cyclic two-level heat engine driven beyond the linear-response regime, determining the corresponding optimal cycle, the maximum power, and the efficiency at maximum power. In Ref. [14] we generalize the previous approach to derive a variational approach to optimal control that can be applied to coherently driven, open quantum dynamical systems.

In Ref. [5] we studied the thermopower of a three-terminal setup composed of a quantum dot attached to three electrodes, one of which is a topological superconductor. We compared the results for s-wave (trivial) and p-wave (topological) superconductors and observed that for small temperatures the thermopower has different sign in the two cases. This behavior is strongly dependent on temperature and we estimate an energy scale that controls the sign in the p-wave case, which results proportional to the square root of the gap and the coupling to superconductor. In Ref. [8] instead we introduce and analyze a class of multi-terminal devices where (electronic) heat and charge currents can follow different paths. We demonstrated that this regime allows to control independently heat and charge flows and to greatly enhance thermoelectric performances at low temperatures. We analyzed in details a three-terminal setup involving a superconducting lead, a normal lead and a voltage probe, showing that in the regime of heat-charge current separation both the power factor and the

figure of merit  $ZT$  are highly increased with respect to a standard two-terminal system. Building up from these effects a magnetic thermal switch for heat management at the nanoscale was then presented in Ref. [9], see Fig. 1.

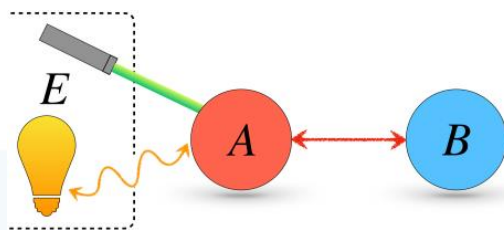


**Figure 1.** Schematic of the operational principle of the magnetic thermal switch discussed in Ref. [9] – figure taken from such article. 1, 2, and 3, represent the contact of the models, while the arrows the heat currents that can be activated by properly changing the external magnetic field.

The Eigenstate Thermalization Hypothesis (ETH), quantum-quenched systems equilibrate towards canonical, thermal ensembles. While at first glance the ETH might seem a very strong hypothesis in Ref. [10] we showed that it is indeed not only sufficient but also necessary for thermalization. More specifically, we considered systems coupled to baths with well-defined macroscopic temperature and show that whenever all product states thermalize then the ETH must hold. Our result definitively settles the question of determining whether a quantum system has a thermal behavior, reducing it to checking whether its Hamiltonian satisfies the ETH.

In Ref. [15] we presented geometrical bounds for the irreversibility of open quantum system in the presence of thermal bath. Entropy production and thermalization effects were discussed in Ref. [16] while in Ref. [22] we consider control through instantaneous Gaussian unitary operations on the ubiquitous lossy channel, and find locally optimal conditions for the cooling and heating of a multimode Gaussian state subject to losses and possibly thermal noise.

In Ref. [17] we investigated a quantum battery made of  $N$  two-level systems, which is charged by an optical mode via an energy-conserving interaction. We quantified the fraction  $E(N)$  of energy stored in the battery that can be extracted in order to perform thermodynamic work demonstrating that  $E(N)$  is highly reduced by the presence of correlations between the charger and the battery or between the two-level systems composing the battery. We then showed that the correlation-induced suppression of extractable energy, however, can be mitigated by preparing the charger in a coherent optical state. We concluded by proving that the charger-battery system is asymptotically free of such locking correlations in the limit of large  $N$ . In Ref. [18] the energy charging of a quantum battery was analyzed in an open quantum setting, where the interaction between the battery element and the external power source is mediated by an ancilla system (the quantum charger) that acts as a controllable switch – see Fig. 2. In Ref. [19] a comparison between quantum and classical battery models was proposed.



**Figure 2.** Schematic of the charger-mediated energy transfer model for quantum battery discussed in Ref. [18]-figure taken from the article. Here B is the battery, A the charger, and E the external energy source.

## References

- [1] R. Alicki, R. Kosloff, *Introduction to quantum thermodynamics: history and prospects. Thermodynamics in the Quantum Regime: Fundamental Aspects and New Directions*, Berlin, Springer (2019).
- [2] J. Goold, M. Huber, A. Riera, L. del Rio, P. Skrzypczyk, *The role of quantum information in thermodynamics-a topical review*, J. Phys. A: Math. Theor. **49**, 143001 (2016).
- [3] A. Mari, A. Farace, V. Giovannetti, *Quantum optomechanical piston engines powered by heat*, Journal of Physics B: Atomic, Molecular and Optical Physics **48 (17)**, 175501 (2015).
- [4] A. Carlini, A. Mari, V. Giovannetti, *Time-optimal thermalization of single-mode Gaussian states*, Phys. Rev. A **90**, 052324 (2014).
- [5] S. Valentini, R. Fazio, V. Giovannetti, F. Taddei, *Thermopower of three-terminal topological superconducting systems*, Phys. Rev. B **91**, 045430 (2015).
- [6] S. Lorenzo et al., *Heat flux and quantum correlations in dissipative cascaded systems*, Phys. Rev. A **91**, 022121 (2015).
- [7] V. Mukherjee et al., *Efficiency of quantum controlled non-Markovian thermalization*, New J. Phys. **17**, 063031 (2015).
- [8] F. Mazza et al., *Separation of heat and charge currents for boosted thermoelectric conversion*, Phys. Rev. B **91**, 245435 (2015).
- [9] R. Bosisio et al., *Magnetic thermal switch for heat management at the nanoscale*, Phys. Rev. B **91**, 205420 (2015).
- [10] G. De Palma et al., *Necessity of Eigenstate Thermalization*, Phys. Rev. Lett. **115**, 220401 (2015).
- [11] V. Cavina, A. Mari, V. Giovannetti, *Optimal processes for probabilistic work extraction beyond the second law*, Scientific Reports **6**, 29282 (2016).
- [12] O. Maillet et al., *Optimal Probabilistic Work Extraction beyond the Free Energy Difference with a Single-Electron Device*, Phys. Rev. Lett. **122**, 150604 (2019).
- [13] V. Cavina, A. Mari, V. Giovannetti, *Slow Dynamics and Thermodynamics of Open Quantum Systems*, Phys. Rev. Lett. **119**, 050601 (2017).
- [14] V. Cavina, A. Mari, A. Carlini, V. Giovannetti, *Optimal thermodynamic control in open quantum systems*, Phys. Rev. A **98**, 012139 (2018).
- [15] L. Mancino et al., *Geometrical Bounds on Irreversibility in Open Quantum Systems*, Phys. Rev. Lett. **121**, 160602 (2018).
- [16] S. Cusumano et al., *Entropy production and asymptotic factorization via thermalization: A collisional model approach*, Phys. Rev. A **98**, 032119 (2018).
- [17] G.M. Andolina et al., *Extractable Work, the Role of Correlations, and Asymptotic Freedom in Quantum Batteries*, Phys. Rev. Lett. **122**, 047702 (2019).
- [18] D. Farina et al., *Charger-mediated energy transfer for quantum batteries: An open-system approach*, Phys. Rev. B **99**, 035421 (2019).
- [19] G.M. Andolina et al., *Quantum versus classical many-body batteries*, Phys. Rev. B **99**, 205437 (2019).
- [20] P. A. Erdman et al., *Maximum power and corresponding efficiency for two-level heat engines and refrigerators: optimality of fast cycles*, New J. Phys. **21**, 103049 (2019).
- [21] P. Abiuso, V. Giovannetti, *Non-Markov enhancement of maximum power for quantum thermal machines*, Phys. Rev. A **99**, 052106 (2019).
- [22] U. Shackerley-Bennett et al., *Locally optimal symplectic control of multimode Gaussian states*, Quantum Sci. Technol. **2**, 044014 (2017).